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20/5

Tugas Mandiri halaman 74

1). Selesaikan PD $y'' - (x-2)y' + 2y = 0$ di sekitar ~~x=~~ $x = 2$

- Transformasikan ke u dengan pemisalan $u = x - a$

$u = x - 2$

$x = u + 2$

Sehingga, PD menjadi : $y'' - uy' + 2y = 0$

- Tentukan $\frac{du}{dx}$

$\frac{du}{dx} = 1$, akibatnya : $y' = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 1 \cdot \frac{dy}{du}$

$y'' = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{d^2} \frac{du}{dx}$

$= \frac{d^2y}{du^2}$

- PD setelah transformasi

$$\frac{d^2y}{du^2} - u\frac{dy}{du} + 2y = 0$$

- Misalkan $y = \sum_{x=0}^{\infty} a_n u^n$

▶ $y = \sum_{n=0}^{\infty} a_n u^n$

▶ $y' = \sum_{n=1}^{\infty} n a_n u^{n-1}$ ▶ $y'' = \sum_{n=2}^{\infty} n(n-1) a_n u^{n-2}$

- Substitusi kembali y, y', dan y'' ke PD awal

$$\sum_{n=2}^{\infty} (n-1)n\, a_n u^{n-2} - u \sum_{n=1}^{\infty} n\, a_n u^{n-1} + 2\sum_{n=0}^{\infty} a_n u^n = 0$$

$$\sum_{n=0}^{\infty} (n+1)(n+2)\, a_{n+2} u^n - \sum_{n=1}^{\infty} n a_n u^n + 2\sum_{n=0}^{\infty} a_n u^n$$

$$\left(\sum_{n=0}^{\infty} \left[(n+1)(n+2)\, a_{n+2} + 2a_n\right] - \sum_{n=1}^{\infty} n\, a_n\right) u^n = 0$$

• $n=0$

$$(n+1)(n+2)\,a_{n+2} + 2a_n$$

$$= 2a_2 + 2a_0$$

Sehingga : $(2a_2 + 2a_0) + \sum_{n=1}^{\infty} \left[(n+2)(n+1)\,a_{n+2} + (2-n)\,a_n \right] u^n = 0$

didapatkan :

•> $n=0$

$$2a_2 + 2a_0 = 0$$

$$a_2 = -a_0$$

•> $n = 1, 2, 3, \ldots$

$$(n+1)(n+2)\,a_{n+2} + (2-n)a_n = 0$$

$$(n+1)(n+2)\,a_{n+2} = (n-2)a_n$$

$$a_{n+2} = \frac{(n-2)\,a_n}{(n+1)(n+2)}$$

Sehingga :

• $n=1$

$$a_3 = -\frac{a_1}{6}$$

• $n=2$

$$a_4 = 0$$

• $n=3$

$$a_5 = \frac{a_3}{20} = -\frac{a_1}{120}$$

• $n=4$

$$a_6 = \frac{2a_4}{30} = 0$$

$n=5$

$$a_7 = \frac{3a_5}{42} = -\frac{a_1}{1680 \cdot 1}$$

• Penyelesaian PD

$$y = \sum_{n=0}^{\infty} a_n u^n$$

$$= a_0 u^0 + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4 + a_5 u^5$$

$$= a_0 + a_1 u + (-a_0)u^2 - \frac{a_1}{6}u^3 + 0 + \left(-\frac{a_1}{120}\right)u^5$$

$$= a_0(1-u^2) + a_1\left(u - \frac{1}{6}u^3 - \frac{u^5}{120} + \ldots\right)$$

$$= a_0(1-u^2) + a_1\left(u - \frac{u^3}{3!} - \frac{u^5}{5!} - \frac{u^7}{1680} - \ldots\right)$$

$$= a_0\left(1-(x-2)^2\right) + a_1\left((x-2) - \frac{(x-2)^3}{3!} - \frac{(x-2)^5}{5!} - \frac{(x-2)^7}{1680}\right)$$

2). Selesaikan PD $y'' - x^2 y' - y = 0$ dengan deret kuasa dalam x

- Misalkan : Anggap penyelesaian PD adalah $y = \sum_{=0}^{\infty} a_n x^n$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1} \qquad y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

- Substitusi y, y', y'' ke PD awal

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - x^2 \sum_{n=1}^{\infty} n a_n x^{n-1} - \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n - \sum_{n=2}^{\infty} (n-1) a_{n-1} x^n - \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=0}^{\infty} \left[(n+2)(n+1) a_{n+2} - a_n\right] x^n - \sum_{n=2}^{\infty} (n-1) a_{n-1} x^n = 0$$

$$\hookrightarrow \underbrace{2a_2 - a_0}_{a_0} + \underbrace{(6a_3 - a_1)}_{a_1} x + \sum_{n=2}^{\infty} \left[(n+2)(n+1) a_{n+2} - a_n - (n-1) a_{n-1}\right] x^n = 0$$

$\circ >$ $n = 0$

$$2a_2 - a_0 = 0$$
$$2a_2 = a_0$$
$$a_2 = \frac{1}{2} a_0$$

$\circ >$ $n = 1$

$$6a_3 - a_1 = 0$$
$$a_3 = \frac{1}{6} a_1$$

$\circ >$ Untuk $n \geq 2$

$$(n+2)(n+1) a_{n+2} - a_n - (n-1) a_{n-1} = 0$$
$$(n+2)(n+1) a_{n+2} = a_n + (n-1) a_{n-1}$$
$$a_{n+2} = \frac{a_n + (n-1) a_{n-1}}{(n+2)(n+1)}$$

▶ Untuk $n = 2$

$$a_4 = \frac{a_2 + a_1}{12} = \frac{1}{24} a_0 + \frac{a_1}{12}$$

▶ Untuk $n = 3$

$$a_5 = \frac{a_3 + 2a_2}{20} = \frac{1}{120} a_1 + \frac{a_0}{20}$$

▶ $n = 4$

$$a_6 = \frac{a_4 + 3a_3}{30} = \frac{a_0}{720} + \frac{a_1}{360} + \frac{a_1}{60} = \frac{a_0 + 2a_1 + 12a_1}{720} = \frac{a_0 + 14a_1}{720}$$

$\circ >$ Diperoleh Persamaan PD

$$y = \sum_{n=0}^{\infty} a_n x^n$$
$$= a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \dots$$
$$= a_0 + a_1 x + \frac{1}{2} a_0 x^2 + \frac{1}{6} a_1 x^3 + \left(\frac{1}{24} a_0 + \frac{1}{12} a_1\right) x^4 + \left(\frac{1}{120} a_1 + \frac{1}{20} a_0\right) x^5 + \dots$$
$$= a_0 \left[1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^5}{20} + \dots\right] + a_1 \left[x + \frac{x^3}{3!} + \frac{x^4}{12} + \frac{x^5}{120} + \dots\right]$$
$$y = a_0 \left[1 + \frac{x^2}{2} + \frac{x^4}{24} + \frac{x^5}{20} + \dots\right] + a_1 \left[x + \frac{x^3}{6} + \frac{x^4}{12} + \frac{x^5}{120} + \dots\right]$$

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Latihan Soal halaman 78

1). $(x^2+4)y'' + y = x$ di sekitar $x = 1$

- Misalkan $u = x-1$ agar dapat dideretkan di $u = 0$

  misal : $u = x-1$

- Tentukan $\frac{dy}{du}$ dan $\frac{d^2y}{du^2}$

$$y' = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{dy}{du}$$

$$y'' = \frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[\frac{dy}{du}\right] \cdot \frac{du}{du}$$

$$= \frac{d^2y}{du^2} \cdot \frac{du}{dx}$$

$$= \frac{d^2y}{du^2}$$

- Persamaan baru PD :

$$(x^2+4) \cdot y'' + y = x$$

$$((u+1)^2+4)\, y'' + y = u+1$$

- Anggap $y = \sum_{n=0}^{\infty} a_n x^n$ penyelesaian PD

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1} \qquad y'' = \sum_{n=2}^{\infty} (n-1)n\, a_n x^{n-2}$$

- Substitusi y' dan y'' ke PD awal

$$(u^2+2u+5) \sum_{n=2}^{\infty} (n-1)n\, a_n u^{n-2} + \sum_{n=0}^{\infty} a_n u^n = u+1$$

$$\sum_{n=2}^{\infty} (n-1)n\, a_n u^n + 2\sum_{n=2}^{\infty} (n-1)n\, a_n u^{n-1} + 5\sum_{n=2}^{\infty} (n-1)n\, a_n u^{n-2} + \sum_{n=0}^{\infty} a_n u^n = u+1$$

$$\sum_{n=2}^{\infty} (n-1)n\, a_n u^n + 2 \cdot \sum_{n=1}^{\infty} n(n+1) a_{n+1} u^n + 5\sum_{n=0}^{\infty} (n+1)(n+2) a_{n+2} u^n + \sum_{n=0}^{\infty} a_n u^n = u+1$$

$$\sum_{n=0}^{\infty} \left[5(n+1)(n+2) a_{n+2} + a_n\right] u^n + \sum_{n=1}^{\infty} \left(2n(n+1) a_{n+1}\right) u^n + \sum_{n=2}^{\infty} (n-1)n\, a_n u^n = u+1$$

$$10a_2 + a_0 + (30a_3 + a_1)u + 4a_2 u + \sum_{n=2}^{\infty}\left[5(n+1)(n+2)a_{n+2} + a_n + 2n(n+1)a_{n+1} + (n-1)n\, a_n\right] u^n = u+1$$

•> Untuk $n = 0$

$$10a_2 + a_0 = 1$$
$$10a_2 = 1 - a_0$$
$$a_2 = \frac{1-a_0}{10}$$

•> Untuk $n = 1$

$$30a_3 + a_1 + 4a_2 = 1$$
$$30a_3 = 1 - a_1 - 4a_2$$
$$= 1 - a_1 - \left(\frac{1}{10} + \frac{a_0}{10}\right)4$$
$$= \frac{10 - 10a_1 - 4 + 4a_0}{10}$$
$$= \frac{4a_0 - 10a_1 + 6}{10}$$
$$a_3 = \frac{4a_0 - 10a_1 + 6}{300}$$
$$= \frac{2a_0 - 5a_1 + 3}{150}$$

•> Untuk $n = 2, 3, \ldots$

$$5(n+2)(n+1)a_{n+2} + a_n + 2(n+1)n\,a_{n+1} + n(n-1)a_n = 0$$
$$5(n+2)(n+1)a_{n+2} = -a_n - 2(n+1)n\,a_{n+1} - n(n-1)a_n$$
$$a_{n+2} = \frac{-a_n - 2(n+1)n\,a_{n+1} - n(n-1)a_n}{5(n+1)(n+2)}$$

• $n = 2$

$$a_4 = \frac{-a_2 - 12a_3 - 2a_2}{60}$$
$$= \frac{-3a_2 - 12a_3}{60}$$
$$= \frac{-3\left(\frac{1-a_0}{10}\right) - 12\left(\frac{4a_0 - 10a_1 + 6}{300}\right)}{60}$$
$$= \frac{\frac{-3-3a_0}{10} - \left(\frac{16a_0 - 40a_1 + 24}{100}\right)}{60}$$
$$= \frac{-30 - 30a_0 - 16a_0 + 40a_1 - 24}{6000}$$
$$= \frac{-54 - 46a_0 + 40a_1}{6000}$$
$$a_4 = \frac{-27 - 23a_0 + 20a_1}{3000}$$

Sehingga, diperoleh PD

$$y = \sum_{n=0}^{\infty} a_n u^n = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4 + \ldots$$
$$= a_0 + a_1 u + \left(\frac{1-a_0}{10}\right)u^2 + \left(\frac{2a_0 - 5a_1 + 3}{150}\right)u^3 + \left(\frac{-27 - 23a_0 + 20a_1}{3000}\right)u^4 + \ldots$$
$$= a_0 + a_1 u + \frac{1}{10}u^2 - \frac{a_0}{10}u^2 + \frac{a_0}{75}u^3 - \frac{a_1}{30}u^3 + \frac{1}{50}u^3 - \frac{27}{3000}u^4 - \frac{23a_0}{3000}u^4 + \frac{20a_1}{3000}u^4 + \ldots$$
$$= a_0\left[1 - \frac{u^2}{10} + \frac{u^3}{75} - \frac{23u^4}{3000} + \ldots\right] + a_1\left[u - \frac{u^3}{30} + \frac{20}{3000}u^4 + \ldots\right] + \frac{1}{10}u^2 + \frac{1}{50}u^3 - \frac{27}{3000}u^4 + \ldots$$
$$= a_0\left[1 - \frac{(x-1)^2}{10} + \frac{(x-1)^3}{75} + \frac{23(x-1)^4}{3000} + \ldots\right] + a_1\left[u - \frac{u^3}{30} + \frac{u^4}{150} + \ldots\right] + \frac{1}{10}u^2 + \frac{1}{50}u^3 - \frac{27}{3000}u^4 + \ldots$$